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Glauber-Model-Based Computational Evaluation of ^{31}Ne Differential Cross Sections for Exploring Halo Phenomena in Neon Nuclei

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ABSTRACT

One-nucleon removal cross section of ^{30}Ne core fragment from ^{31}Ne + ^{12}C reaction system was computed as a sum of the elastic and inelastic reaction cross sections in the frame work of the Glauber Theory using the CSC_GM code. The neon nucleus $^{31}_{10}Ne$ which consists of ten protons and twenty one neutrons having a low binding energy is expected to be a neutron halo candidate. The projectile nucleus is assumed to have ^{30}Ne + ^{1}n structure. Both the contributions from the elastic and inelastic reaction cross sections were shown. The contribution from the inelastic break up cross section is much larger than that from the elastic cross section in the energy region beyond a few hundred MeV per nucleon, the wide gap between $(P+T)$ and $(C+T)$ and also the large nucleon removal cross section of 137.32mb at projectile energy of 900MeV suggest ^{31}Ne to have a neutron halo structure. Which are found to agree with the experimental data.

Keywords: *Glauber model, One-nucleon halo, Reaction Cross Section.*

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1.0 Introduction

Nuclear halo formation is an important feature of nuclei with extreme neutron to proton number asymmetry near the limits of nuclear stability. Halo nuclei are very weakly-bound exotic states of nuclear matter in which the outer one or two valence nucleons (usually neutrons) are spatially decoupled from a relatively tight bound core such that they spend more than half their time beyond the range of the binding nuclear potential.

The term halo is defined, according to (Al-khalili, 2004), as a *threshold* effect arising from the very weak binding of the last one or two valence nucleons (usually neutrons) and hence decoupling from, a well-defined inert 'core' containing all the other nucleons.

Necessary conditions for halo formation are that one or two valence neutrons have: (a) weak binding, with typical separation energies *Sn* <1 MeV, which are significantly lower than the more conventional 8 MeV, and (b) low orbital angular momentum, $\ell = 0$ or 1 (Sawhney, et., al, 2014). All halo nuclei have the same features of having a large nuclear matter radius (far beyond the nuclear-interaction range) and low binding energies of valence nucleons surrounding the core. Based on these characteristics, the structural properties of highly neutron rich halo nuclei lying near the neutron drip line has attracted the interests of many scientists and researchers all over the world.

Investigation of nuclear halo structure can be done by reproducing experimental reaction observables such as Reaction Cross Sections, Coulomb dissociation cross sections and momentum distributions following nuclear breakup (Adamu (a), 2013). These quantities play important role in revealing the nuclear structure of unstable nuclei, particularly the halo structure, proton and neutron skins.

Different models have been used to investigate the halo nature of unstable radioactive nuclei, by calculating the reaction observables. These include CDCC, Adiabatic and Glauber models (Yahiro, et., al 2012), (Al-khalili, 2004), (Al-khalili & Nunes, 2003).

The nucleus ^{31}Ne, which has 21 neutrons (N = 21), is also an interesting candidate as it was predicted to be among those that settled within the so-called island of inversion. ^{31}Ne which was first predicted by (Descouvemont, 1999) in the projectile fragmentation reaction. The reaction cross sections σ_R of Ne isotopes on ^{12}C target at 240 MeV/nucleon (Ahmad et., al, 2015) and the one-neutron removal cross sections σ_{-1n} of ^{31}Ne on ^{12}C (Sharma and Patra 2012) and ^{208}Pb targets have been measured for the first time around 230 MeV/nucleon (Nakamura et., al, 2009). Also Calculation of Reaction cross section of Ne isotopes on ^{12}C at 240 MeV/nucleon within the framework of Glauber Model (Ahmad, 2016), which involved the oscillator model for the densities of the colliding nuclei. In order to describe the tail of the last neutron of ^{31}Ne, (Minomi et., al, 2012) quantitatively analyses the reaction cross sections of $^{28–32}$Ne by ^{12}C at 240 MeV/nucleon, using the double-folding model. The work partly summarized above concentrate on the calculation of reaction cross section at fixed energy, so there is need to calculate this quantity at different energies.

In this paper one-nucleon removal cross section of ^{30}Ne core fragment from the $^{31}Ne + {}^{12}C$ reaction system was calculated in the framework of the Glauber Theory using the CSC_GM code. The reaction system is described as $^{31}Ne + {}^{12}C \rightarrow {}^{12}C + {}^{30}Ne + n$: The projectile nucleus (^{31}Ne) is assumed to have the structure of a core nucleus (^{30}Ne) plus a valence neutron. Measurement of one-nucleon removal cross section is one of the standard works for the study of unstable nuclei which provide detail information about nuclear structure.

The Glauber model is a microscopic reaction theory of high energy collision based on the eikonal approximation and on the bare nucleon-nucleon interaction (*B. Abu-Ibrahim et., al* 2003). It has become a standard tool to calculate the one-nucleon removal cross section because it can account for a significant part of breakup effects which play an important role in the reaction of a weakly bound nucleus (Esha, 2012).

2.0 Research Background

Considering the reaction of a projectile nucleus P with a target nucleus T. At the initial stage of the reaction, the projectile in the ground state described with an intrinsic wave function ψ_o impinges with momentum $\hbar K$= (0, 0, $\hbar K$) on the target in its ground state described with an intrinsic wave function Θ_c. The center-of-mass wavefunction is removed from ψ_a (Θ_0). At the final stage of the reaction, the projectile goes to the state *a* specified by a wave function ψ_a and the target goes to the state *c* specified by a wave function Θ_c . The state *a* is not necessarily a bound state but may be a continuum state that includes some fragments. The momentum transferred from the target to the projectile is $\hbar \boldsymbol{q}$, (Tanihata, 1996), (Ozawa, et., al, 2001).

The scattering amplitude for this reaction is written in the Glauber theory as an integral over the impact parameter $\boldsymbol{b}$ between the projectile and the target (Adamu and Sa'adu, 2018), (Abu-ibrahim and Suzuki, 2002) as

$$F_{ac}(q) = \frac{i\kappa}{2\pi}\int d\boldsymbol{b} e^{-\mathbf{q}.\mathbf{b}} \langle \psi_a \Theta_c | 1 - \prod_{i\in P}\prod_{j\in T}(1-\Gamma_{ij}) | \psi_o \Theta_0 \rangle \quad (1)$$

The integrated cross section for this reaction is given by

$$\sigma_{ac} = \int \frac{dq}{\kappa^2} |F_{ac}(\boldsymbol{q})|^2 \quad (2)$$

The profile function in eq. (1) is determined so as to fit empirical nucleon-nucleon scattering amplitudes and is taken as follows

$$\Gamma(\boldsymbol{b}) = \frac{1-i\alpha}{4\Pi\beta}\sigma_{NN} e^{-b^2/2\beta} \quad (3)$$

The parameters σ_{NN}, α and β usually depend on either the proton–proton (neutron–neutron) or proton–neutron case, but we assume that some appropriate average values are used in this calculations. The argument of Γ_{ij} in Eq. (1) is $\boldsymbol{b} + \boldsymbol{s}_i^P - \boldsymbol{s} - \boldsymbol{s}_j^T$ which stands for the impact parameter between *ith* and *jth* nucleons. Here $\boldsymbol{s}_i^P$ ($\boldsymbol{s}_i^T$) is the two-dimensional coordinates comprising the *x*- and *y*-components of the *ith* nucleon coordinate in the projectile (target) relative to its center-of-mass coordinate.

2.1 Total reaction cross section

The total reaction cross section is obtained by summing σ_{ac} over all possible final states *ac* except for the elastic Channel (Ogawa, et al., 2003) and is given as

$$\sigma_{reac}(P+T) = \sum_{ac\neq 00}\sigma_{ac} \quad (4)$$

The projectile-target total cross section is given by

$$\sigma_{reac}(P+T) = \int db \left(1 - \left|e^{i\chi_{PT}(b)}\right|^2\right) \quad (5)$$

Where the phase-shift function χ_{TP} is given by

$$e^{i\chi_{PT}(b)} = \langle \psi_o \Theta_0 \left| \prod_{i\in P} \prod_{j\in T} \left[1 - \Gamma\left(b + s_i^P - s_j^T\right)\right] \right| \psi_o \Theta_0 \rangle \quad (6)$$

Also the core-target total reaction cross section is given by

$$\sigma_{reac}(C + T) = \int db \left(1 - \left|e^{i\chi_{CT}(b)}\right|^2\right) \quad (7)$$

2.2 One-nucleon removal cross section

The one-nucleon removal cross section σ_{-N} is separated into elastic and inelastic cases (Shukla, el., at 2014).

The cross section due to the elastic breakup process is given by

$$\sigma_{-N}^{el} = \int db \left\{ \langle \varphi_0 \left| e^{-2Im\chi_{CT}(b_C) - 2Im\chi_{NT}(b_C+s)} \right| \varphi_0 \rangle - \left| \langle \varphi_0 \left| e^{i\chi_{CT}(b_C) + \chi_{NT}(b_C+s)} \right| \varphi_0 \rangle \right|^2 \right\} \quad (8)$$

While the cross section from inelastic breakup is given by

$$\sigma_{-N}^{inel} = \int db \, \langle \varphi_0 \left| e^{-2Im\chi_{CT}(b_C)} - e^{-2Im\chi_{CT}(b_C - s)} \right| \varphi_0 \rangle \quad (9)$$

The one-nucleon removal cross section for halo nuclei is approximately equal to the difference between the two reaction cross section,

$$\sigma_{reac}(P + T) - \sigma_{reac}(C + T) \quad (10)$$

The core-target phase-shift function, χCT and the nucleon-target phase-shift function, χNT are defined through the relevant densities ρ by (Ogawa et al., 2003)

$$i\chi CT = -\int \mathrm{d}\boldsymbol{r} \int \mathrm{d}r' \rho C(\boldsymbol{r}) \rho T(\boldsymbol{r}') \Gamma(\boldsymbol{b} + \boldsymbol{s} - \boldsymbol{s}'), \quad (11)$$

$$i\chi NT(\boldsymbol{b}) - \int \mathrm{d}\boldsymbol{r} \, \rho T(\boldsymbol{r}) \Gamma(\boldsymbol{b} - \boldsymbol{s}) \quad (12)$$

The density ρ is normalized to the mass number of a nucleus, $\int \mathrm{d}r \, \rho(\boldsymbol{r}) = \mathbf{A}$, and it is assumed to be given by a combination of Gaussians: (Shukla et al., 2014), (Sharma, Panda, Sharma, & Patra, 2013).

$$\rho(r) = \sum_i c_i \, \mathrm{e}^{-a_i r^2} \quad (13)$$

Where c_i is Gaussian coefficient to fit the nuclei density and a_i is the range.

The projectile is assumed to be a system of a core nucleus coupled with a valence nucleon. Its ground-state wave function is given by

$$\Psi_0 = \varphi_0 \Phi_0 \quad (14)$$

where Φ_0,is the ground state wave function of the core, and φ_0 is the valence-nucleon wave function.

3.0 Methodology

The CSC_GM code (Cross Section Calculations in the Glauber Model), is a Fortran 90 program, which is made up of three categories of files: The physics source code which is the main source code that contains the routine for the actual computations. The input files contain data to be read into the main program at run-time. The output files keep the results of the

computation. The code used for calculating various types of reactions cross sections consisting of a core plus one valence-nucleon system, was slightly modified to enable the computation of one-nucleon removal cross section of ^{30}Ne core fragment in the ^{31}Ne + ^{12}C reaction system.

The input data specify the reaction system, the type of cross section, the target and core densities, and the control parameters of the Metropolis algorithm etc. The input data for the ^{31}Ne + ^{12}C reaction system are read from an input file csc.inp. The configuration of the csc.inp file is as follows: The first line gives the mass numbers of the target, projectile and core (A_T,A_P, and A_C), the second line gives the charge numbers of those nuclei (Z_T, Z_T, and Z_C). This code assumes, $A_P - A_C = 1$. The third line defines the incident energy of the projectile per nucleon (in MeV). The fourth line define the parameters of the nucleon–nucleon profile function, σ_{NN} (in fm^2), α, and β (in fm^2). The fifth line gives the orbital angular momentum of the valence nucleon. The sixth line specifies the condition for the Monte Carlo quadrature, the number of configuration points (N_S), the step size (δ in fm) for the random walk in the Metropolis algorithm, and the seed number for generating random numbers (irand). The seventh line is the control parameter for the single-particle wave function of the valence nucleon. The eighth line gives the number of Gaussians used to fit the target and the core densities. The ninth lines give the coefficients $c_i$and the ranges α_i of the Target density. The last line gives the coefficients $c_i$and the ranges α_i of the core density (in fm-^{2}). Results of the computations are written on a file *react.out*. The multi-dimensional integration over the valence nucleon coordinates is performed with a Monte Carlo technique. In the Monte Carlo integration, a set of configuration or integration points is generated according to a suitably chosen guiding function ***w(x)***. The random walk with the Metropolis algorithm is taken to generate such points. The code generates the single particle wave function from the parameters. The radial part of the single-particle wave function $R_{nlj}(r) = ru_{nlj}(r)$, which is used as the guiding function ***w(x)*** is obtained by solving a Schrödinger equation,

$$\frac{d^2R(r)}{dr^2} + \frac{2\mu}{\hbar^2}\left[E - U(r) - \frac{l(l+1)\hbar^2}{2\mu r^2}\right]R(r) = 0 \qquad (15)$$

with a potential U (r)

$$U(r) = -V_0 f(r) + V_{ls}(l.s){r_0}^2 \frac{1}{r}\frac{d}{dr}f(r) + V_{coul}, \qquad (16)$$

Where $f(r) = \left[1 + \exp\frac{r-R}{\alpha}\right]^{-1}$ with $R = r_0 A_C^{1/3}$,

where $r_0$and α are the radius and diffuseness parameters in fm respectively. $V_0$is the initial depth of the potential. $V_{ls} = 17$ MeV (Ogawa et al., 2003).

4.0 Results and Discussion

The input parameters filled in the files *csc.inp* and *wf.inp* are shown in Tables 1and 2.

Table 1: *csc.inp* input file for ^{31}Ne + ^{12}C reaction system.

S/N	INPUT PARAMETERS	VALUES
1	Mass numbers of target, projectile and core: (A_T; A_P; A_C)	12, 31, 30
2	Atomic numbers of target, projectile and core: (Z_T; Z_P; Z_C)	6, 10, 10
3	Incident Energy per nucleon (in MeV)	900
4	Profile function parameters (σ_{NN}, α and β in fm^2)	4.17, -0.21, 0.200
5	l (angular momentum quantum number)	0
6	Monte Carlo parameters (Ns, δ, irand)	500000, 2.5, −11213
7	*icond1*(1:available, 0: not available)	1
8	Number of Gaussians used to fit the core and target densities	2
9	Coefficient c_i, rangea_i(in fm^{-2}) of the Target	-1.25421 0.39907 1.39728 0.355553
10	Coefficient c_i rangea_i(in fm^{-2}) of the Core	-2.902931 0.22326 3.048814 0.2047651

Table 2: wf.inp input file for **^{31}Ne + ^{12}C** reaction system.

S/N	INPUT PARAMETERS	VALUES
1	Initial depth V_0 of the optical potential (in MeV)	47.41
2	Diffuseness parameter (in fm) a	0.75
3	Radius parameter r_0 (in fm)	1.25
4	Energy eigenvalue for the valence nucleon (in MeV)	0.31
5	j value for the valence nucleon orbit	1.5
6	Node number for the valence nucleon orbit	1

Table 3. The sample output for **^{31}Ne + ^{12}C**reaction system at 900MeV.

S/N	Cross Section (in mb)	Result
1	Projectile reaction cross section	1642.29
2	Core reaction cross section	1454.42
3	Nucleon removal cross section	187.08
4	N-removal (elastic)	49.76
5	N-removal (Inelastic)	137.32

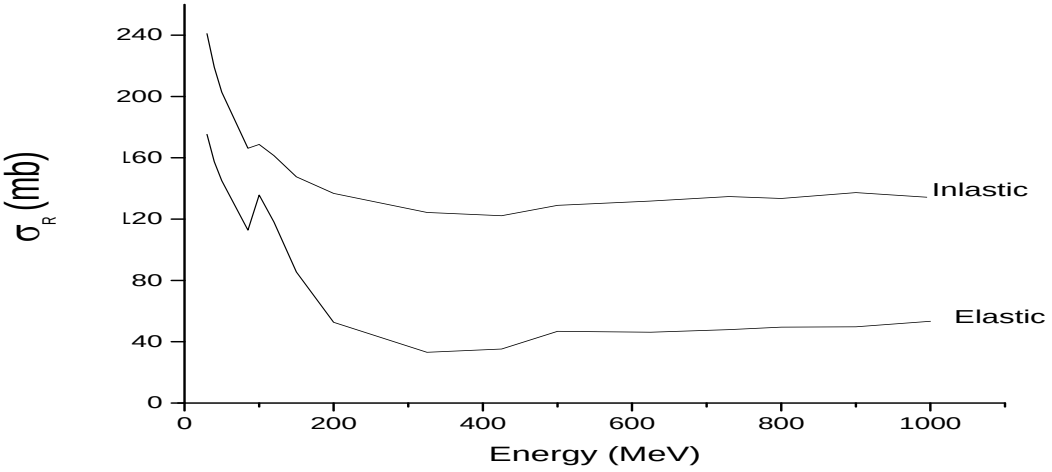


Figure 1. Graph of Elastic and Inelastic Differential Cross Sections

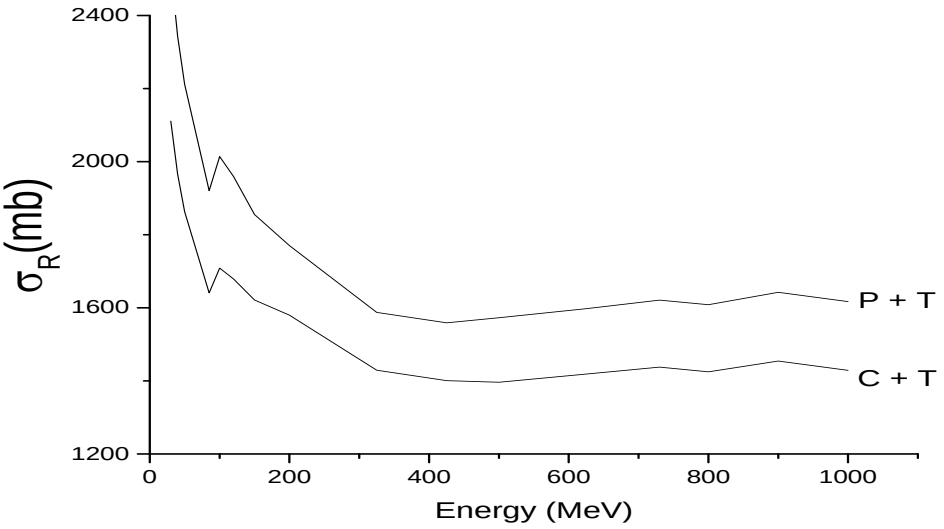


Figure 2. Graph of One Nucleon Removal Cross Sections

5.0 Discussion

The computations of one-nucleon removal cross section of ^{31}Ne core fragment from ^{31}Ne + ^{12}C reaction system in the framework of the Glauber theory was done using the CSC_GM code. The ground state of ^{31}Ne is assumed to be ^{30}Ne + one neutron system. The core and target densities used to fit the Gaussians, the density, and profile function parameters were taking from (Sharma, et al., 2013), (Shukla et al., 2014), (El-din & Hassan, 2010). The valence neutron is in the $1P_{2/3}$ orbit with n = 1, l = 2, j = 1.5, and the separation energy ε=0.33 MeV (Horiuchi et.,al 2010). The one-nucleon removal cross section in ^{31}Ne+^{12}C is large reflecting the halo structure. For example, at incident energy of 900MeV the elastic and inelastic parts of the one-nucleon removal cross section are 49.76mb and 137.32mb respectively, and the total removal cross section is 187.08mb as shown in table 3. The higher value of this cross section corresponds to the low binding energy for the last neutron in ^{31}Ne nucleus.

6.0 Conclusion

The wider gap between elastic and inelastic removal cross sections occur in the energy region beyond a few hundred MeV per nucleon as shown in fig. 1. This indicated that Glauber model is appropriately valid at high energy. In conclusion, the results suggest that this approach should be considered as a good tool to calculate reaction cross section observables.

7.0 Recommendations

After the research has been done, the following recommendations were made:

a.) The use of computerized programs which can be done at home for conducting research without going to laboratories was recommended.
b.) Government should prioritize sponsoring these kind of researches which can be achieved with a little fund.
c.) This research was carried out for only one nucleon halo nuclei, so researchers should embark in finding a program that will compute for two nucleon halo nuclei.

8.0 Acknowledgement

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