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CYLINDRICAL AND SPHERICAL ION-ACOUSTIC SOLITARY WAVES IN A DUSTY PLASMA WITH NON-THERMAL ELECTRONS

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ABSTRACT

The propagation characteristics of nonlinear ion acoustic solitary waves are investigated in a non-planar plasma with fluid ions, stationary dust particles, and non-thermal electrons. By using the reductive perturbation technique (RPT), we were able to derive the Korteweg-de Vries (KdV) and modified Korteweg-de Vries (mKdV) equations. It is demonstrated that the non-planar Kdv equation may be converted into a planar geometry using the proper coordinate transformation. It has been found that the combined effects of dust and non-thermal electrons have a significant impact on the non-planar geometry ion acoustic nonlinear solitary wave structures. The joint impact of non-thermal electrons and dust on the solitary wave has been well examined. With a decrease in τ, the soliton amplitude rises. The solitons are dependent on the β (non-thermal parameter) and α (the ratio of equilibrium values of electron and ion number densities). Solitary wave structures created are both compressive and rarefactive.

Key words: Ion-acoustic, Solitary waves, Dusty plasma, Non-thermal electrons

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INTRODUCTION

In recent years, research into the analysis of nonlinear solitary formations in various types of plasmas has attracted a lot of attention from academics. Many studies have been published on nonlinear structures, including solitary, vortices, cnoidal waves, etc. with various charged particle velocity distributions (Maxwellian/non-Maxwellian) (Sagdeev, 1966; Saini et al., 2013; Sultana & Mamun, 2014; Tribeche et al., 2012; Washimi & Taniuti, 1996; Yadav et al., 1995).

The presence of dust particles in astrophysical environments, such as planetary rings, cometary tails, asteroid zones, the earth's ionosphere, mesosphere, and laboratory apparatus, has been documented in the literature (Shukla and Mamun, 2002). Massive dust particles make up dusty plasma, which materially alters the dynamics of electron-ion plasma (Mendis& Rosenberg, 1994). It offers new freedoms that result in new behaviours.

Dust ion-acoustic waves are produced when charged dust particles are present in electron-ion plasma. These waves can be found in the earth's magnetosphere at the plasma sheet boundary layer and the polar cap boundary layer. It was theoretically and empirically proven that these dust ion-acoustic solitary waves exist (Barkan et al., 1995; Shukla & Silin, 1992). It has been established through many investigations that the distributions of electrons and ions are essential for understanding the characteristics of nonlinear wave structures. The distribution functions of the extremely non-thermal electrons have been measured in numerous investigations, indicating the presence of very energetic electrons (Goldman et al., 1999).Freja and Viking satellites both reported non-thermal distributions of electrons in space plasmas (Bostrom, 1992; Doyner et al., 1994). Many studies using non-thermal electrons have been published to investigate the numerous nonlinear processes in plasma (Berbri & Tribeche, 2009; Chatterjee et al., 2009; Saha & Chatterjee, 2009).

It has been proposed that the characteristics of solitary structures are altered by the presence of non-thermal distribution of electrons (Caims et al., 1995). The effects of non-thermal electrons and negatively charged ions on ion acoustic solitary waves contained in a finite geometry were investigated by Bhattacharya et al. in 2003. The pseudo-potential approach was used in this. It was discovered that the wave's stability is significantly influenced by the bounded plasma's finite shape. After that, the single wave's structure was examined in relation to the parameter that gauges its departure from the stabilized state. Both compressive and rarefactive solitons are observed when one takes into account non-thermal electrons and negative ions. In a non-thermal dusty plasma, Kaur and Saini (2017) studied the periodic ion-acoustic waves. It has been found that electron non-thermality has a significant impact on periodic wave shapes.

A typical nonlinear hyperbolic system can be reduced using the reductive perturbation technique (RPT) to a single solvable nonlinear equation that describes the system's far field. It is a method for creating streamlined models that describe the interaction and propagation of nonlinear waves. According to Taniuti (1974), it reduces long waves to the Burgers equation or the Kortweg-de Vries (KdV) equation. Over the years, a great deal of research has been done on the Kortweg-de Vries (KdV) or modified Kortweg-de Vries (KdV) equation. In a thought-provoking manner. However the majority of research is restricted to one-dimensional geometry. Mamun and Shukla, in 2002, investigated the characteristics of cylindrical and spherical dust ion-acoustic solitary waves in an un-magnetized dusty plasma made up of stationary dust particles, inertial ions, and Boltzmann electrons.

The modified Korteweg-de Vries equation was derived and its numerical solutions were obtained using the reductive perturbation approach. In contrast to those in a one-dimensional geometry, it was discovered that the properties of the DIASWs in a non-planar cylindrical or spherical geometry are different. Another un-magnetized dusty plasma with static negatively charged dust fluid, non-thermally distributed electrons, and an adiabatically ionized fluid was eventually taken into consideration by (Akpabio & Antia, 2015). The Burgers' equation for non-planar geometry has been derived using the fundamental characteristics of the dust-ion-acoustic shock waves. Numerical analysis of the modified Burgers' equation solution in non-planar geometry revealed that the non-planar geometry effects are crucial to the formation of shock waves. They also found that the shock wave profile is greatly altered when the non-thermal electron dispersion is taken into account. The Korteweg-de Vries equation and modified Korteweg-de Vries equation will be derived in this study, and we'll next demonstrate how the solitons behave in the cylindrical geometry.

THEORETICAL ANALYSIS

The following normalized set of ion fluid equations govern the axially (radially) symmetric dynamics of an un-magnetized collision-less plasma made up of inertial ions, negatively charged dust, and non-thermal electrons.

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial r}(n\mathrm{v})+\frac{\upsilon}{r}(n\mathrm{v})=0 \qquad (1)$$

$$\frac{\partial \mathrm{v}}{\partial t}+\mathrm{v}\frac{\partial \mathrm{v}}{\partial r}+\frac{\partial \phi}{\partial r}=0 \qquad (2)$$

$$\frac{\partial^2 \phi}{\partial r^2}+\frac{\upsilon}{r}\frac{\partial \phi}{\partial r}=n_e\alpha-n+(1-\alpha) \qquad (3)$$

where υ is the radial velocity component in the cylindrical ($\upsilon=1$) and spherical ($\upsilon=2$) coordinates, n is the ion number density normalized by its equivalent value n_0, n_e is the electron number density,v is the ion fluid velocity normalized to ion-acoustic speed $C=Temi12$, ϕ is the electrostatic wave potential normalized to $e\phi Te$. The time(t) variable and space (r) variable are in units of ion plasma period ω_p^{-1} and the Debye length $\lambda_D=\sqrt{T_e/4\pi n_0 e^2}$ respectively. The dust particle $\delta_d=(1-\alpha)$, the ratio of the equilibrium values of electron and ion number densities is denoted by $\alpha={n_{e0}}/{n_0}$. It should be noted that $0<\alpha<1$. When $\alpha>1$ the effect of the negatively charged dust grain on dust ion acoustic wave is negligible. The normalized non-thermal electron density is given as:

$$n_e=(1-\beta\phi+\beta\phi^2)e^{\phi} \qquad (4)$$

Where β is the non-thermal parameter. For Maxwellian distribution, $\beta = 0$. Expanding n_e of (4) using Taylor series expansion and substituting it into (3), we obtain

$$\frac{\partial^2\phi}{\partial r^2} + \frac{\upsilon}{r}\frac{\partial\phi}{\partial r} = \alpha\left[1 + \phi(1-\beta) + \frac{\phi^2}{2}\right] - n + (1-\alpha) \tag{5}$$

To derive the KdV equation, we employed the reductive perturbation method and introduced the following stretched coordinates

$$\zeta = \epsilon^{1/2}(r - \lambda t) \quad \text{and} \quad \tau = \epsilon^{3/2}t \tag{6}$$

whereϵ is the smallness parameter measuring the weakness of nonlinearity and dispersion, λ is the phase velocity of dust ion acoustic waves. Expanding the field variables into a power series of the parameter ϵ as:

$$\begin{aligned} n &= 1 + \epsilon(n_1 + \epsilon n_2 + \epsilon^2 n_3 + \cdots) \\ \mathrm{v} &= \epsilon(\mathrm{v}_1 + \epsilon \mathrm{v}_2 + \epsilon^2 \mathrm{v}_3 + \cdots) \\ \phi &= \epsilon(\phi + \epsilon\phi_2 + \epsilon^2\phi_3 + \cdots) \end{aligned} \tag{7}$$

Substituting (6) and (7) into (1), (2) and (5); and collecting the coefficients of different powers of ϵ we obtain the following sets of equations:

To lowest order:

$$\begin{aligned} &-\lambda\frac{\partial n_1}{\partial\zeta} + \frac{\partial \mathrm{v}_1}{\partial\zeta} = 0 \\ &n_1 = \alpha\phi_1(1-\beta) \\ &-\lambda\frac{\partial \mathrm{v}_1}{\partial\zeta} + \frac{\partial\phi_1}{\partial\zeta} = 0 \end{aligned} \tag{8}$$

To next higher order:

$$\begin{aligned} -\lambda\frac{\partial n_2}{\partial\zeta} + \frac{\partial \mathrm{v}_2}{\partial\zeta} + \frac{\partial n_1}{\partial\tau} + \frac{\upsilon \mathrm{v}_1}{\epsilon\zeta + \lambda\tau} + \frac{\partial}{\partial\zeta}(n_1\mathrm{v}_1) = 0 \\ -\lambda\frac{\partial \mathrm{v}_2}{\partial\zeta} + \frac{\partial\phi_2}{\partial\zeta} + \frac{\partial \mathrm{v}_1}{\partial\tau} + \mathrm{v}_1\frac{\partial \mathrm{v}_1}{\partial\zeta} = 0 \\ n_2 = -\frac{\partial^2\phi_1}{\partial\zeta^2} + \alpha\phi_2(1-\beta) + \alpha\frac{\phi_1^2}{2} \end{aligned} \tag{9}$$

From (8), we obtained the following equation.

$$\begin{aligned} \phi_1 &= \varphi(\zeta,\tau) \\ v_1 &= \alpha^{1/2}\varphi_1(1-\beta)^{1/2} \\ n_1 &= \alpha\varphi_1(1-\beta) \\ \lambda &= \frac{1}{\alpha^{1/2}(1-\beta)^{1/2}} \end{aligned} \tag{10}$$

Substituting (10) into (9) we obtain:

$$\begin{gathered} -\lambda\frac{\partial n_2}{\partial\zeta} + \frac{\partial v_2}{\partial\zeta} + \alpha(1-\beta)\frac{\partial\varphi_1}{\partial\tau} + \frac{\upsilon\alpha\varphi_1(1-\beta)}{\tau} + \alpha^{3/2}(1-\beta)^{3/2}\frac{\partial(\varphi_1^2)}{\partial\zeta} = 0 \\ -\lambda\frac{\partial v_2}{\partial\zeta} + \frac{\partial\varphi_2}{\partial\zeta} + \alpha^{1/2}(1-\beta)^{1/2}\frac{\partial\varphi_1}{\partial\tau} + \alpha(1-\beta)\varphi_1\frac{\partial\varphi_1}{\partial\zeta} = 0 \\ n_2 = -\frac{\partial^2\varphi_1}{\partial\zeta^2} + \alpha\varphi_2(1-\beta) + \alpha\frac{\varphi_1^2}{2} \end{gathered} \tag{11}$$

Eliminating the φ_2, n_2,v_2 between the set equations in (11) and making necessary substitutions, the KdV equation of cylindrical and spherical ion acoustic waves in non-thermal dusty plasma is obtained as

$$\frac{\partial\varphi_1}{\partial\tau} + \frac{\upsilon\varphi_1}{2\tau} + A\varphi_1\frac{\partial\varphi_1}{\partial\zeta} + B\frac{\partial^3\varphi_1}{\partial\zeta^3} = 0 \tag{12}$$

where nonlinear coefficient (A) and dispersion coefficient (B) are given as

$$A = \frac{1}{2[\alpha(1-\beta)]^{1/2}}\left[3\alpha(1-\beta) - \frac{1}{(1-\beta)}\right] \tag{13}$$

and

$$B = \frac{1}{2[\alpha(1-\beta)]^{3/2}} \tag{14}$$

As seen in (13), it is a function of $\alpha$and β. Depending on the values of α and β the coefficient A maybe positive or negative. It is seen from the expression of A when $\beta = 0$ and $\alpha = 1/3$ the coefficient A vanishes and the cylindrical and spherical KdV equation collapses into the linear KdV equation. By using new stretched coordinates, we consider the modified cylindrical and spherical KdV in order to balance the nonlinearity with dispersion:

$$\zeta = \epsilon(r - \lambda t), \qquad \tau = \epsilon^3 t \tag{15}$$

Following the same procedures, for lower power:

$$-\lambda\frac{\partial n_1}{\partial\zeta} + \frac{\partial v_1}{\partial\zeta} = 0 \tag{16}$$

$$-\lambda \frac{\partial \mathrm{v}_1}{\partial \zeta} + \frac{\partial \phi_1}{\partial \zeta} = 0$$
$$n_1 = \alpha \phi_1 (1 - \beta)$$

For next higher power:

$$-\lambda \frac{\partial n_2}{\partial \zeta} + \frac{\partial \mathrm{v}_2}{\partial \zeta} + \frac{\partial n_1}{\partial \tau} + \frac{\partial}{\partial \zeta}(n_1 \mathrm{v}_1) = 0$$
$$-\lambda \frac{\partial \mathrm{v}_2}{\partial \zeta} + \frac{\partial \phi_2}{\partial \zeta} + \mathrm{v}_1 \frac{\partial \mathrm{v}_1}{\partial \zeta} = 0 \qquad (17)$$
$$n_2 = \alpha \phi_2 (1 - \beta) + \alpha \frac{\phi_1^2}{2}$$

For next higher power:

$$-\lambda \frac{\partial n_3}{\partial \zeta} + \frac{\partial \mathrm{v}_3}{\partial \zeta} + \frac{\partial n_1}{\partial \tau} + \frac{\upsilon \mathrm{v}_1}{\lambda \tau} + \frac{\partial}{\partial \zeta}(n_1 \mathrm{v}_2 + n_2 \mathrm{v}_1) = 0$$
$$-\lambda \frac{\partial \mathrm{v}_3}{\partial \zeta} + \frac{\partial \phi_3}{\partial \zeta} + \frac{\partial \mathrm{v}_1}{\partial \tau} + \frac{\partial (\mathrm{v}_1 \mathrm{v}_2)}{\partial \zeta} = 0 \qquad (18)$$
$$n_3 = -\frac{\partial^2 \phi_1}{\partial \zeta^2} + \alpha(\phi_3(1 - \beta) + \phi_1 \phi_2) + \frac{1}{6}\phi_1^3$$

Eliminating the quantities φ_2, $\varphi_3 n_2$, $n_3 \mathrm{v}_2$,$\mathrm{v}_3$between the set (18), by making necessary substitutions, the following equation is obtained

$$\frac{\partial \varphi_1}{\partial \tau} + \frac{\upsilon \varphi_1}{2\tau} + C\varphi_1^2 \frac{\partial \varphi_1}{\partial \zeta} + B\frac{\partial^3 \varphi_1}{\partial \zeta^3} = 0 \qquad (19)$$

where nonlinear coefficient $(C)$is given as

$$C = \frac{3}{2[\alpha(1-\beta)]^{1/2}}\left[\frac{\alpha}{2} + 2\alpha^2(1-\beta)^2 - \frac{1}{6(1-\beta)}\right] \qquad (20)$$

(19) is the modified Korteweg-de Vries (mKdV) equation of cylindrical and spherical ion acoustic solitary waves in non-thermal dusty plasma.

(21) provides the exact solution for spherical and cylindrical KdV applying the appropriate transformation (Kaur & Bala, 2017; Sahu & Roychoudhury, 2003).

$$\phi_1 = \frac{1}{\tau}\left\{\frac{\zeta}{2A} + \left(\frac{3u}{A}\right) sech^2\left[\sqrt{\frac{u}{4B\tau}}(\zeta + 2u)\right]\right\} \qquad (21)$$

where u is the soliton velocity.

RESULTS AND DISCUSSION

The resulting cylindrical and spherical Kdv equations and their solutions will be numerically examined in this section. We took the stationary solution of (12) into consideration as the starting point. The answer can be summed up as shown below. We considered the stationary solution of (12), without the term$\upsilon\varphi_1/_{2\tau}$, as the initial condition. The solution takes the following form:

$$\varphi_1 = \varphi_0 sech^2\left[\frac{(\zeta - u\tau)}{\omega}\right] \tag{22}$$

Where the amplitude and thickness of the solitons are $\varphi_0 = {}^{3u}/_{A}$ and $\omega = 2\sqrt{{}^{B}/_{u}}$ respectively. Numerical solution For large values of τ, the term ${}^{\upsilon\varphi_1}/_{2\tau}$ is no longer dominant in our equation, making the cylindrical and spherical KdV similar to one dimensional solitary wave. However, as the values of τ decreases, the term ${}^{\upsilon\varphi_1}/_{2\tau}$ becomes dominant, and both spherical and cylindrical solitary waves differ from one dimensional solitary wave.

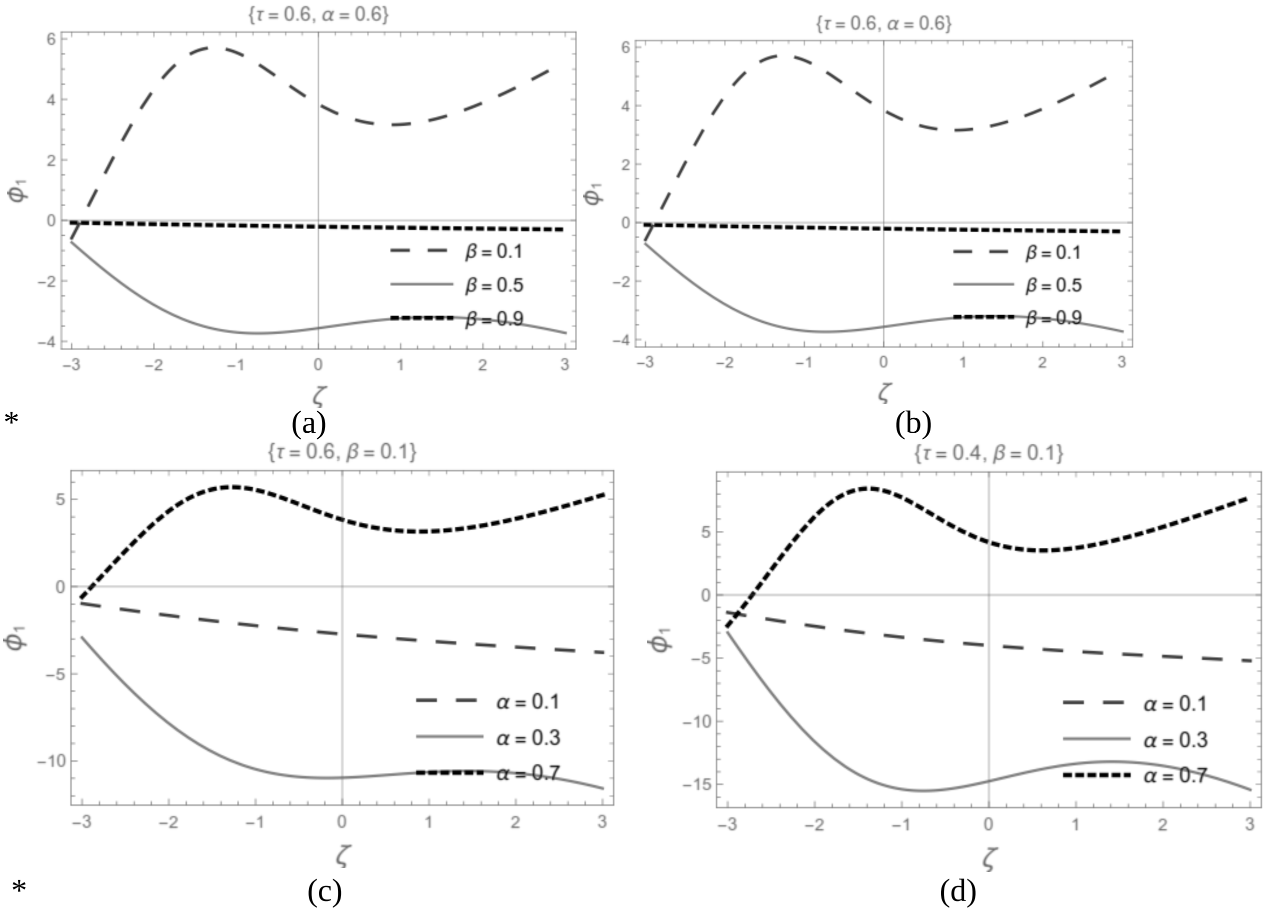

Figure 1: Plot of ϕ_1 against ζ for cylindrical KdV equation with $u = 0.8$, and various values of time τ, β and α.

Figure 1 shows the behaviour of the nonlinear term coefficient of the Kdv equation with respect to non-thermal electrons. According to Fig. 1a, for fixed values of β, α, and $u = 0.8$, as τ increases, the wave amplitude decreases. These gradually affect the nonlinearity of the soliton wave as it diminishes gradually.

From Figures 1b, 1c and 1d, the graphs have similar behaviours. These figures show the dependency of the amplitude and width of DIA solitons on the β (non-thermal parameter) and α (the ratio of equilibrium values of electron and ion number densities). As a result, the solitons are significantly impacted by α, τ, and β,. The wave appears to be compressive or rarefactive at certain key values of β and α. Moreover, the soliton wave's nonlinearity can varnish at some values. In terms of physics, this indicates that the nonlinearity is minor in comparison to the dispersive effects. A modified Kdv equation is required when the nonlinearity of the wave fades because the cylindrical Kdv equation degenerates into a linear

Kdv equation.

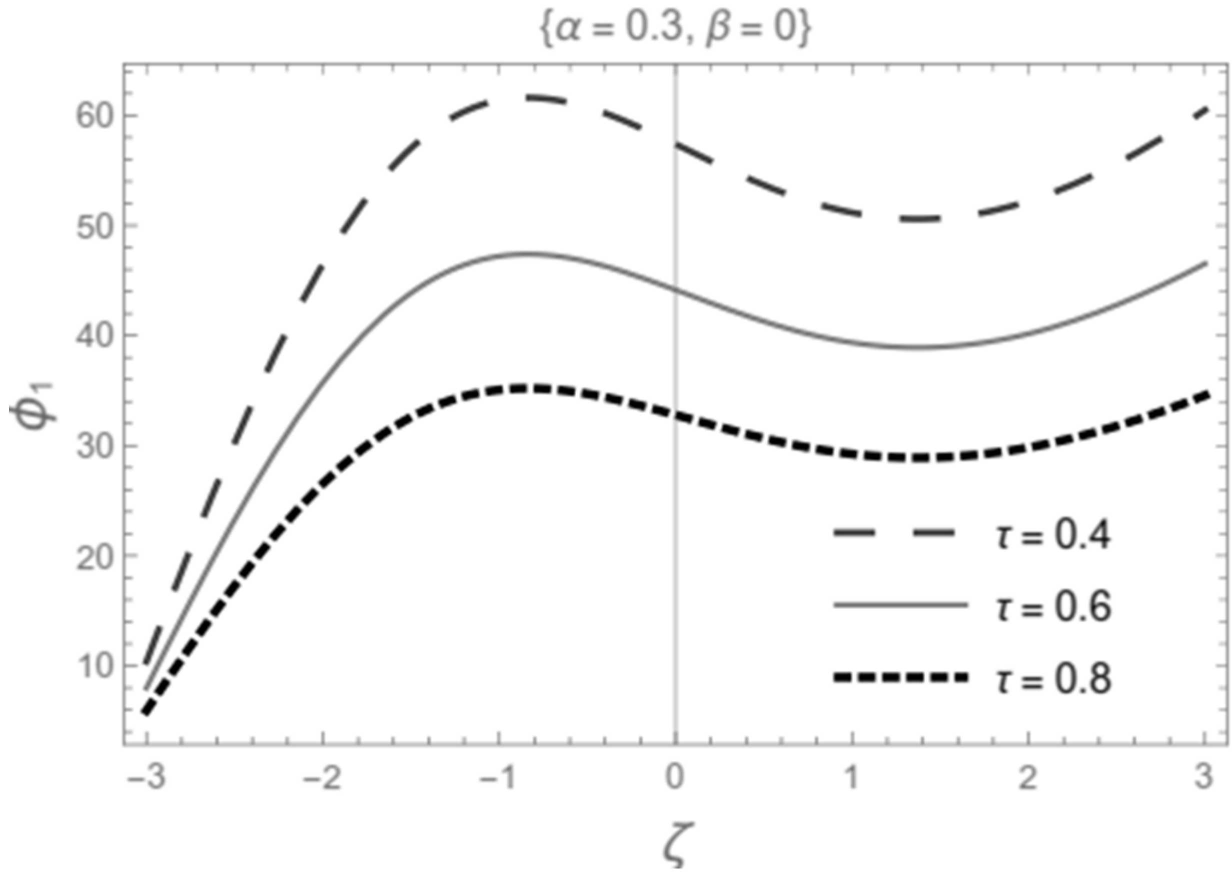

Figure 2: Plot of ϕ_1 against ζ for cylindrical mKdV equation with $\beta = 0$, $\alpha = 0.3$, $u = 0.8$ and various values of τ.

Figure 2 shows the plot of ϕ_1 against ζ of (21) for cylindrical mKdV equation with $\beta = 0$, $\alpha = 0.3$, $u = 0.8$ and various values of time τ. When$\beta = 0$, $\alpha = 0.3$ the linear Kdv equation degenerates from the cylindrical Kdv equation. The modified Kdv was created to address such a situation. As shown in Figure 1a, the wave amplitude decreases as τ increases. This also demonstrates the validity of our mKdv equation.

CONCLUSION

In the current study, we looked at how ion acoustic nonlinear solitary waves behave in an un-magnetized plasma made up of fluid ions, immobile negative particles, and electrons that follow non-thermal distribution while taking non-planar geometry into account. KdV equation has been generated and examined using the reductive perturbation technique. It was discovered that:

i. The cylindrical and spherical KdV resemble a one-dimensional solitary wave for large values of τ because the term $^{\upsilon\varphi_1}/_{2\tau}$ is no longer dominating in our equation. As the value of τ decreases, however, the term $^{\upsilon\varphi_1}/_{2\tau}$ becomes dominant, and both spherical and cylindrical solitary waves vary from one dimensional solitary wave.
ii. With a drop in τ, the soliton amplitude increases.

iii. Both amplitude and width of DIA solitons are dependent on the β (non-thermal parameter) and α(the ratio of equilibrium values of electron and ion number densities).

iv. Critical values of β(non-thermal parameter) and α(the ratio of equilibrium values of electron and ion number densities) can lead to the formation of compressive and rarefactive solitary structures.

The findings of the current study may be applicable to astrophysical plasma environments, where dust coexists with ions and non-thermal electrons.

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